Gaussian Beam Optical Systems with High Gain or High Loss Media

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***Abstract*—Coherent electromagnetic wave amplifiers with nonnegligible gain per wavelength are included in the Gaussian beam matrix formalism and a procedure is developed for propagating Gaussian beams in optical systems that may include unsaturated amplifiers and similar absorbers. Standard formulas for beam spotsize and radius of curvature in a uniform medium are generalized in a new way to include gain or loss. An asymmetric focal shift and a potentially infinite spotsize are predicted. These dramatic effects are interpreted physically.**

I. Introduction

GAUSSIAN beam theory was popularized by Kogelnik who showed that, like paraxial light rays, Gaussian beams could be traced through a wide variety of optical systems by simple 2 x 2 matrix multiplication [1]. In these systems the light beam remains Gaussian, and a unified systems approach that includes polynomial-Gaussian beams was developed. It is of practical importance to note that many conventional lasers are examples of such optical systems, and hence the output beam modes of these lasers and masers can be simply described by Hermite-Gaussian [2]–[4] or Laguerre-Gaussian [5]–[7] functions. For optical systems with misalignments, the 2 x 2 matrix theory can be generalized to an only slightly more complicated 3 x 3 matrix theory [8].

There are four explicit approximations made in Gaussian beam theory: the "scalar approximation," the "slow spatial variation" approximation, the "paraxial approximation," and the "low gain per wavelength approximation." The scalar approximation and the slow spatial variation approximation involve the spatial variation of the material properties of the medium in which the light beam is propagating [9]–[12]. The paraxial approximation is concerned with the variation of the electromagnetic fields [4], [13]. It may under certain conditions be avoidable [14], [15]. The low gain (or loss) per wavelength approximation is used to ignore the diffractive effects of space-independent gain (or loss) in studying propagation of the beam profile in complex media treating such amplifiers (or absorbers) as perfect dielectrics. These four approximations are independent, and an important example of this independence is a steady-state beam in a gain-focused laser. In this case, the spotsize and radius of curvature of the beam do not change with distance, and thus no paraxial approximation needs to be made. This situation exists regardless of the level of gain or loss.

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The universally-employed low-gain (low-loss) approximation cans be avoided in linear media. Thus, the purpose of this study is to include simple amplifiers and absorbers in the beam matrix formalism, and to examine how Gaussian beam parameters vary in these media as compared to lossless dielectric media. It is found that the effects of the active medium are important when either the gain (or loss) per wavelength is large or the light beam travels many Rayleigh lengths inside the active material. In this case, the distance traveled per Rayleigh length per gain per wavelength may approach unity. Of course, the gain per wavelength may be large if either the amplifying medium possesses a significant incremental gain or if the wavelength is large as in the case of microwave amplification. Dye lasers are commonly operated with a thin dye jet amplifier where the incremental gain is large. Semiconductor lasers may also possess a large incremental gain, and wavelength-sized laser oscillators have been built [16], [17]. The demonstration of single-atom masers also shows the possibility of very high gain per wavelength [18]. It is not surprising that the transverse modes of these high gain lasers may vary drastically from their low gain counterparts, since other properties such as oscillation frequency are known to vary significantly as well [19]. High loss per wavelength can also occur in many absorbing media such as dyes or metals, and the propagation of the Gaussian beam parameters in such media is examined here for the first time.

In Section II, the Gaussian beam matrix is found for flat and spherical boundaries between two linear media which may possess gain or loss, and the Gaussian beam matrix for a thin lens of such material is written. Though several of the matrices derived herein have similar forms to well-known matrix representations for optical elements, they are complex-valued to account for gain or loss and their applications and implications may be different. As in other treatments, it is assumed that the active medium does not saturate, and hence higher order effects such as spatial and spectral holeburning and cross-relaxation are ignored. However, the formalism may include amplifier dispersion and may be used to account for such effects as asymmetry in the frequency spectrum [20], [21], mode pulling and mode splitting [22], and chirped pulse amplification [23], [24]. In Section III, formulas for spotsize and radius of curvature of Gaussian beams in spatially homogeneous active media are found, and an asymmetric focal shift is predicted. Similar focal shifts

TABLE I
GAUSSIAN BEAM MATRIX REPRESENTATIONS FOR SEVERAL OPTICAL ELEMENTS

Optical Medium	Schematic	Beam Matrix
Uniform Medium	d; 1, 2	$\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$
Lossless Thin Lens	f	$\begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}$
Spherical Mirror	R	$\begin{bmatrix} 1 & 0 \\ -2/R & 1 \end{bmatrix}$
Thin Cornercube		$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
Spherical Interface	k_{o1}, k_{o2}, R	$\begin{bmatrix} 1 & 0 \\ \frac{1}{R}\left[1-\frac{k_{o1}}{k_{o2}}\right] & \frac{k_{o1}}{k_{o2}} \end{bmatrix}$
Axial Medium $k=k_o(z)$	d; 1, 2	$\begin{bmatrix} 1 & k_o(0)\int_0^d k_o^{-1}(z')dz' \\ 0 & k_o(0)/k_o(d) \end{bmatrix}$
Lenslike Medium $k=k_o-\frac{1}{2}k_2x^2$	d; 1, 2	$\begin{bmatrix} \cos(k_2^{1/2}d/k_o^{1/2}) & (k_0^{1/2}/k_2^{1/2})\sin(k_2^{1/2}d/k_o^{1/2}) \\ -(k_2^{1/2}/k_o^{1/2})\sin(k_2^{1/2}d/k_o^{1/2}) & \cos(k_2^{1/2}d/k_o^{1/2}) \end{bmatrix}$
Gaussian Aperture	w_a	$\begin{bmatrix} 1 & 0 \\ -i\lambda/(\pi w_a^2) & 1 \end{bmatrix}$
Active Thin Lens	f, α_o	$\begin{bmatrix} 1 & 0 \\ -1/f-i\lambda\alpha_o/[2\pi(n-1)f] & 1 \end{bmatrix}$

have been observed experimentally in a uniform absorber [25]. In these experiments, asymmetric focal shifts were achieved using 1053 nm, 1 ps laser pulses in an argon vapor. Standard formulas for beam spotsize and radius of curvature in a uniform medium are generalized in a new way to include gain or loss. It is shown that the spotsize error using the conventional method can approach infinity in a finite distance.

II. GAUSSIAN BEAM MATRICES FOR HIGH GAIN MEDIA

In this section the beam matrix for a flat boundary between active media is derived and is generalized to spherical (actually parabolic) boundaries. Combining spherical boundaries results in the matrix for a thin lens of active material, and a corresponding "active" lensmaker's formula is obtained. The results are a group of complex-valued matrices which are organized into Table I. Because of the use of beam matrices, previous methods for investigating the mode stability [26], [27] and synthesis [28], [29] of Gaussian beam optical systems that include the matrices derived herein still apply.

The parameter most commonly traced through a Gaussian beam optical system is the complex quantity $Q/k_o = 1/q$ where k_o represents the complex propagation constant of the medium. Hence, if the medium changes, then so will Q/k_o. For purposes of this study, media in which k_o is purely real are referred to as dielectric or passive, while those with nonzero imaginary part are referred to as complex or active - this includes amplifiers and absorbers. On a given plane, the real and imaginary parts of the beam parameter Q are related to the physical parameters of the nearly plane wave

$$Q_r = \frac{2\pi n}{\lambda R} \tag{1a}$$

$$Q_i = -\frac{2}{w^2}. \tag{1b}$$

The symbol w, known as "spotsize," is the radius of the Gaussian beam where the electric field amplitude has decreased to $1/e$ of its maximum. The radius of curvature R, of the phase fronts at a given point on the propagation axis is defined to be positive when the center of curvature of the phase fronts is behind that point. The symbol n is the index of refraction of the medium in which the nearly plane wave is propagating. The symbol λ is the wavelength of a plane wave propagating in freespace having the same frequency as the Gaussian beam. The quantity Q/k_o can now be related to the Gaussian beam's spotsize and radius of curvature by the relation

$$\frac{1}{q} \equiv \frac{Q}{k_o} = \frac{\beta_o/R - i2/w^2}{\beta_o + i\alpha_o} = \frac{[1/R - 2\alpha_\lambda/(\beta_0 w^2)] - i[\alpha_\lambda/R + 2/(\beta_o w^2)]}{1+\alpha_\lambda^2} \tag{2}$$

where the real and imaginary parts of the axial propagation constant k_o are β_o and α_o respectively. For nearly plane waves, the axial wavenumber is $\beta_o = 2\pi n/\lambda$, and α_o is the axial exponential electric field gain constant. The normalized incremental gain $\alpha_\lambda \equiv \alpha_o/\beta_o$ is dimensionless. Media with positive α_λ are amplifiers while those with negative α_λ are absorbers. Typically, $|\alpha_\lambda| \ll 1$ and (2) reduces to the usual Gaussian beam relation [1]

$$\frac{1}{q} = \frac{Q}{\beta_o} = \frac{1}{R} - i\frac{\lambda}{n\pi w^2}. \tag{3}$$

However, there may be cases where $\alpha_\lambda \ll 1$ is a poor approximation, and it is our purpose to investigate such scenarios.

The first matrix to be considered is that for a flat boundary between active media. Across a boundary, Kogelnik's "ABCD Law" [1] (also called the "Kogelnik Transformation") is

$$\frac{Q_2}{k_{o2}} = \frac{Q_1}{k_{01}}\frac{k_{01}}{k_{02}} = \frac{C + DQ_1/k_{01}}{A + BQ_1/k_{01}}. \tag{4}$$

This result is a consequence of the constancy of the complex beam parameter, Q, across a flat boundary. It follows from (4) that a possible beam matrix for a flat active boundary is

$$T_{flat\ boundary} = \begin{pmatrix} 1 & 0 \\ 0 & k_{o1}/k_{o2} \end{pmatrix}. \tag{5}$$

This reduces to the well-known boundary matrix for a dielectric medium in the special case that k_{o1} and k_{o2} are real. With this matrix one is able to trace Gaussian beams of light described by (2) through complex paraxial optical systems. One need only apply the matrix (5) with (4) as one would the matrix for a passive optical boundary. However, inside an active medium, the form of (2) implies new expressions for spotsize and radius of curvature as α_λ is nonzero. Of particular interest is that amplifiers can be included in laser resonators, where except for the exponential gain factor, they are usually implicitly neglected.

The matrix for a curved dielectric boundary is

$$T_{spherical\ boundary} = \begin{pmatrix} 1 & 0 \\ \left[1 - \frac{k_{o1}}{k_{o2}}\right] \frac{1}{R} & \frac{k_{o1}}{k_{o2}} \end{pmatrix} \tag{6}$$

where this R is the radius of curvature of the boundary, which is defined to be positive when it is concave to an incident light beam. Again, this reduces to the familiar matrix for a spherical dielectric boundary when k_{o1} and k_{o2} are purely real.

As a practical example, the beam matrix for a thin lens which inevitably has loss (or gain) is found. It may be obtained by cascading two spherical boundaries. The resulting matrix is identical in form to the matrix for a conventional thin lens if the focal length is identified by the relationship

$$\frac{-1}{f_{active}} = (k_{o,eff} - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]. \tag{7}$$

This is the complex generalization of the traditional lens-maker's formula. The symbol f_{active} represents an effective "complex focal length" of the lens and $k_{o,eff}$ is the complex propagation constant of the lens material divided by the complex propagation constant of the medium containing the lens. An even more general formula could be derived in cases where the lens acts as a boundary between two different types of media as, for example, some microscope lenses do. If the background medium is vacuum, then the beam matrix for an active thin lens can be written

$$T_{thin\ lens} = \begin{pmatrix} 1 & 0 \\ \left[-\frac{1}{f} - i\frac{\lambda}{\pi[2(n_o - 1)f/\alpha_o]}\right] & 1 \end{pmatrix} \tag{8}$$

where f would be the focal length of the lens if it were neither amplifying nor absorbing. Comparing (8) and the matrix for a Gaussian aperture [30] it follows that the effective aperture of the lens is

$$w_a{}^2 = 2(n_o - 1)f/\alpha_o. \tag{9}$$

If the active thin lens is positive and amplifying, then from (9) $w_a^2 > 0$ and it acts as if it were a passive thin lens combined with a stabilizing Gaussian aperture. This is reasonable since the lens is thicker on axis and therefore has more gain on axis. Likewise, a thin complex lenslike medium with more gain in the center is known to act as a Gaussian aperture [30]. Conversely, if the active thin lens were positive but absorbing, then it would act as to increase the spotsize of the input beam. Thus, an absorber in the shape of a negative lens would act as a stabilizing aperture.

Media with only axial variations of index of refraction and gain (or loss) are of interest, and the beam matrix is

$$\begin{bmatrix} 1 & k_o(0)\int_0^d k_o^{-1}(z')dz' \\ 0 & k_o(0)/k_o(d) \end{bmatrix}. \tag{10}$$

As with the previous matrices, though the form of (10) is similar to the previously derived real ray matrix [31], this beam matrix is complex.

With the matrices derived in this section, the spotsize and radius of curvature can be traced through Gaussian beam optical systems and resonators that include active media. The beam matrix (5) can be combined with the beam matrix for a homogeneous medium to obtain the transfer characteristics of such a material

$$T_{active\ medium} = \begin{pmatrix} 1 & 0 \\ 0 & k_o/\beta_{air} \end{pmatrix}\begin{pmatrix} 1 & z \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & \beta_{air}/k_o \end{pmatrix} = \begin{pmatrix} 1 & \beta_{air}z/k_o \\ 0 & 1 \end{pmatrix}. \tag{11}$$

The quantity $\beta_{air} = 2\pi/\lambda$ is the wavenumber of freespace. It appears that just as the active thin lens generalizes to a thin lens of "complex focal length," an active homogeneous medium may be thought of as a uniform medium of "complex length." There may be occasions when the matrix (11) being complex is undesirable. This may be the case if, for example, the output beam modes of an optical system are required to be Hermite–Gaussian modes of real argument. The active medium can be cascaded with an optical subsystem so that the total system matrix contains only real elements. One possible choice of such an optical subsystem is represented by (40) in [28]. The subsystem contains lenses and Gaussian apertures whose parameters must be chosen in such a way as to obtain the desired effect.

III. Gaussian Beams in Uniform Amplifiers with High Gain

The purpose of this section is to use the formalism developed in the Section II to investigate Gaussian beam propagation in high gain spatially homogeneous amplifiers and high loss spatially homogeneous absorbers. In particular, the emphasis here is placed on the spotsize and radius of curvature of a Gaussian beam propagating through these media. Comparisons are made between high gain results and their corresponding low gain limits, and cases where the effect of the gain (or loss) is significant are considered.

Since information about the beam spotsize and radius of curvature is contained in the beam parameter, it is appropriate to begin with the Kogelnik Transformation

$$\frac{Q_2}{k_{o2}} = \frac{C + DQ_1/k_{o1}}{A + BQ_1/k_{o1}}. \tag{12}$$

There are several ways to proceed. If the input beam and the output beam are both within the amplifier then k_{o1} and k_{o2} are complex. In this case the beam matrix for an amplifier or absorber is identical to the beam matrix for freespace. Alternately, both the input and the output could be just outside the amplifier. For simplicity, this latter method is used here and the medium outside the amplifier is chosen to be freespace. Thus the transfer characteristics of the medium are of interest and the ABCD matrix is given by (11). Substituting (11) into (12) results in

$$Q_2 = \frac{Q_1 k_o}{k_o + zQ_1}. \tag{13}$$

For simplicity and convention, the coordinate origin ($z = 0$) is chosen at the input , and the input beam is constrained to have flat phase fronts (infinite radius of curvature). In freespace this is also where the spotsize is minimum, and this input plane is called the "waist." Since the input beam has infinite radius of curvature, it follows from (1a) that the real part of the input complex beam parameter is zero ($Q_{1r} = 0$). As before, $k_o \equiv \beta_o + i\alpha_o$, and Q_2 can be written in terms of real and imaginary parts

$$Q_2 = Q_{1i}\left[\frac{-z/z_o + i[1 + \alpha_\lambda(\alpha_\lambda - z/z_o)]}{1 + (z/z_o - \alpha_\lambda)^2}\right]. \tag{14}$$

The quantity $z_o \equiv \beta_o w_1^2/2$ is the usual Rayleigh length which incorporates the medium's refractive index, and α_λ is, as before, the ratio of the exponential gain factor α_o to the wavenumber β_o. The complex beam parameter is related to the Gaussian beam's spotsize and radius of curvature through (3). Substituting (3) into (14) and separating real and imaginary parts yields

$$\frac{w_2^2}{w_1^2} = \frac{1 + (z/z_o - \alpha_\lambda)^2}{1 - \alpha_\lambda(z/z_o - \alpha_\lambda)} \tag{15}$$

$$R_2 = z\left[(1 - \alpha_\lambda z_o/z)^2 + (z_o/z)^2\right]. \tag{16}$$

These two equations are the fundamental results of the section. Their ramifications are examined below.

Equations (15) and (16) are valid for amplifiers ($\alpha_\lambda > 0$), absorbers ($\alpha_\lambda < 0$), and dielectrics ($\alpha_\lambda = 0$). For beam propagation through pure dielectrics, (15) and (16) reduce to the usual formulas for spotsize and radius of curvature

$$\frac{w_2^2}{w_1^2} = 1 + (z/z_o)^2 \tag{17}$$

$$R_2 = z\left[1 + (z_o/z)^2\right]. \tag{18}$$

Beam propagation through amplifiers is now considered. In stark contrast to the conventional spotsize formula (17), a notable feature of (15) is that in a finite distance, the spotsize approaches infinity. This occurs if the length of the amplifier is

$$z_\infty = \frac{1 + \alpha_\lambda^2}{\alpha_\lambda} z_o. \tag{19}$$

If the amplifier is longer than this "catastrophic length," then from (15), the square of the spotsize goes negative, and the light beam, rather than being Gaussian, has a minimum on axis. This suggests a possible mechanism for ring mode formation. Because of this catastrophe effect, it is difficult to define a diffraction angle. Another important result of (15) is additional focusing in amplifiers. It can be shown by minimizing (15) with respect to z that the light beam achieves a minimum

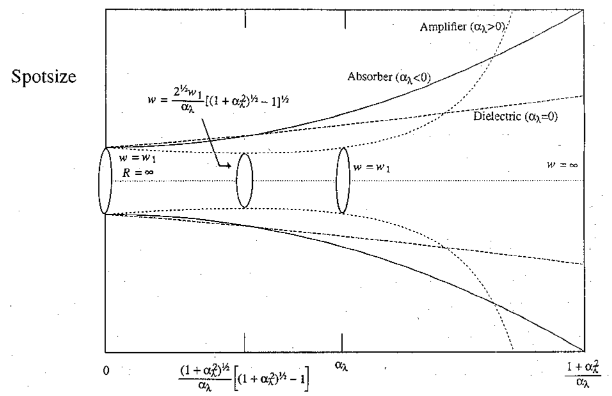

Fig. 1. Beam propagation in uniform amplifiers, absorbers, and dielectrics.

spotsize when

$$z_{focus} = \frac{\left(1 + \alpha_\lambda^2\right)^{\frac{1}{2}}}{\alpha_\lambda}\left[\left(1 + \alpha_\lambda^2\right)^{\frac{1}{2}} - 1\right] z_o \tag{20}$$

and has the corresponding spotsize

$$w_{focus} = \frac{2^{\frac{1}{2}} w_1}{\alpha_\lambda}\left[\left(1 + \alpha_\lambda^2\right)^{\frac{1}{2}} - 1\right]^{\frac{1}{2}}. \tag{21}$$

In the limit of small α_λ, (20) and (21) reduce to $z_{focus} = 0$ and $w_{focus} = w_1$. When $z = \alpha_\lambda z_o$, the beam has returned to its initial size ($w = w_1$) but the radius of curvature is z_o/α_λ. The radius of curvature at the catastrophic length, z_∞ , is this same z_o/α_λ. The radius of curvature remains positive as it propagates through the amplifier, even though the amplitude focuses. The minimum radius of curvature occurs after the focus at $z_o\left(1 + \alpha_\lambda^2\right)^{\frac{1}{2}}$. In an absorber, the spotsize increases monotonically. These beam spotsize results are summarized in Fig. 1. The spotsize of a beam in an amplifier at $-z$ is the same as that for an absorber at $+z$ [i.e., $w(z \to -z, \alpha_\lambda \to -\alpha_\lambda) = w(z, \alpha_\lambda)$].

To understand the catastrophic behavior of the Gaussian beam, the difference between a "nearly plane wave" and a "nearly spherical wave" is examined. The exponential portion of a spherical wave can be written

$$E_{spherical\ wave} = \exp\left[-ik_o R\right] \tag{22}$$

$$= \exp\left[-ik_o\left(x^2 + y^2 + z^2\right)^{\frac{1}{2}}\right] \tag{23}$$

$$\approx \exp\left[-ik_o z\right]\exp\left[-ik_o\frac{x^2 + y^2}{2R}\right] \tag{24}$$

$$= \exp[\alpha_o z]\exp[-i\beta_o z]\exp\left[\alpha_o\frac{x^2+y^2}{2R}\right]\exp\left[-i\beta_o\frac{x^2+y^2}{2R}\right]. \tag{25}$$

Similarly, a Gaussian beam can be written

$$E_{nearly\, plane\, wave} = \exp[\alpha_o z]\exp[-i\beta_o z] \exp\left[-\frac{x^2+y^2}{w^2}\right]\exp\left[-i\beta_o\frac{x^2+y^2}{2R}\right] \tag{26}$$

$$= E_{spherical\ wave} \exp\left[-(x^2+y^2)\left(\frac{1}{w^2}+\frac{\alpha_o}{2R}\right)\right]. \quad (27)$$

Therefore, a Gaussian beam can be written in the usual way as a "nearly plane wave" (26) or as a "nearly spherical wave" (27). One can convert from one formulation to the other if the following identification is made

$$\frac{1}{w^2_{sphere}} = \frac{1}{w^2_{plane}} + \frac{\alpha_o}{2R}. \quad (28)$$

Thus with high gain or loss per wavelength, the amplitude distribution depends sensitively on the shape of the surface over which it is measured, and the spotsize measured on a plane will be different from the spotsize measured on a spherical wave front. In particular, if $\alpha_o/(2R) > 0$ then the spotsize on the spherical wavefront will be smaller than that measured on the corresponding plane. Equation (15) can be rewritten for spotsize on a spherical wavefront by combining it with (16) and (28) to yield

$$w^2_{2,sphere} = \frac{w^2_{1,sphere}}{1+\alpha^2_\lambda}\left[1+(z/z_o-\alpha_\lambda)^2\right]. \quad (29)$$

This is similar to the formula for the spotsize in freespace (17) except that the input beam is slightly scaled and the waist is displaced. Since the denominator is positive definite, the catastrophic behavior is absent on the Gaussian beam's spherical wavefront.

The catastrophic behavior is now examined. On its spherical wavefronts, the Gaussian beam's spotsize is governed by (29). Since the shape of the amplifier output plane is flat and the shape of the last phase front is spherical, there is a mismatch. At the flat output plane of the amplifier, the center of the positive spherical phase front has reached the output, while the sides of the beam undergo additional amplification. This is equivalent to an inverted Gaussian aperture at the output. From (28) it follows that the width of this effective aperture is $w_a^2 = -2R/\alpha_o$. In agreement with our interpretation, the closer the radius of curvature to infinity (the output shape of the amplifier) and the smaller the gain coefficient, the larger the effective aperture and the smaller the effect. It is this effective inverted aperture at the output that causes the catastrophe effect.

Some insight into the initial focusing and defocusing effects may be obtained by further examination of the phase fronts of the beam. For the special case here, the initial phase front is flat. Diffraction causes the next phase front a wavelength into the material to be slightly curved positively. Thus the center of the phase front has gone farther than the sides of the phase front. In the case of an absorber, there is more loss at the center than at the sides and the beam sees an effective loss profile causing defocusing. This is also the case for succeeding phase fronts, and the beam defocuses monotonically. In an amplifier, the farther going center of the phase front sees more gain on-axis and therefore sees an effective gain profile causing the beam to focus.

It can be seen that the catastrophe effect does not exist in absorbers. However, there exists an "inverse catastrophe effect." Thus an inverted Gaussian beam may be converted to a Gaussian beam and it may be possible for the absorber to approximately convert a ring mode to a Gaussian beam. This suggests that there may be complex optical systems (including laser oscillators) where the output spotsize is finite but at one or more intermediate planes within the optical system the spotsize is infinite. Under these conditions care must be taken to insure that all system apertures have been appropriately accounted for.

It is interesting to consider "spotsize error" in a high gain laser amplifier due to the use of the low gain approximation. In Fig. 1, this amounts to dividing the "amplifier" curve by the "dielectric" curve. It can be seen that spotsize error due to the low gain approximation is significant if the length of the amplifier is within an order of magnitude of its catastrophic length, and approaches infinity as the length of the amplifier approaches its catastrophic length.

IV. Conclusion

A general formalism has been developed to propagate Gaussian beams of light in optical systems and resonators representable by complex ABCD Gaussian beam matrices including media with large values of loss or gain. It is found that the usual low gain per wavelength approximation is unnecessary, and Gaussian beam propagation in spatially homogeneous amplifiers and absorbers has been examined.

It is reasonable that the low gain approximation is suspect in media with high loss (or gain) per wavelength. However, a basic result here is that the loss per wavelength can be relatively small, but if the Gaussian beam travels many Rayleigh lengths in the absorber (or amplifier), it is not appropriate to use the low gain approximation as accumulated error can be substantial. These effects may be important in practice since it is not uncommon for absorbers to be thousands of Rayleigh lengths long. This is particularly true in high intensity applications where very strong focusing often occurs.

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